

Anomaly Detection for OOD and Novel Category Detection

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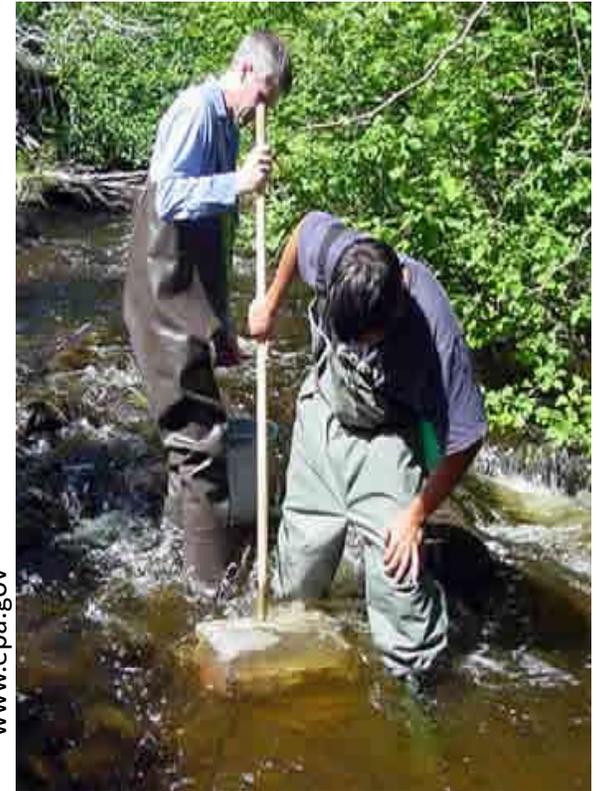
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Faculty members: Alan Fern, Debashis Mondal

Thank you Dan Hendrycks and Balaji Lakshminarayanan for advice and suggestions

Motivating Example: Automated Counting of Freshwater Macroinvertebrates

- Goal: Assess the health of freshwater streams
- Method:
 - Collect specimens via kicknet
 - Photograph in the lab
 - Classify to genus and species
- BugID Project
 - 54 classes of interest to the EPA
 - accuracy \approx 90%
 - Larios, N., Soran, B., Shapiro, L., Martínez-Muños, G., Lin, J., Dietterich, T. G. (2010). **Haar Random Forest Features and SVM Spatial Matching Kernel for Stonefly Species Identification.** *IEEE International Conference on Pattern Recognition (ICPR-2010)*.
 - Lin, J., Larios, N., Lytle, D., Moldenke, A., Paasch, R., Shapiro, L., Todorovic, S., Dietterich, T. (2011). **Fine-Grained Recognition for Arthropod Field Surveys: Three Image Collections.** *First Workshop on Fine-Grained Visual Categorization (CVPR-2011)*
 - Lytle, D. A., Martínez-Muñoz, G., Zhang, W., Larios, N., Shapiro, L., Paasch, R., Moldenke, A., Mortensen, E. A., Todorovic, S., Dietterich, T. G. (2010). **Automated processing and identification of benthic invertebrate samples.** *Journal of the North American Benthological Society*, 29(3), 867-874.



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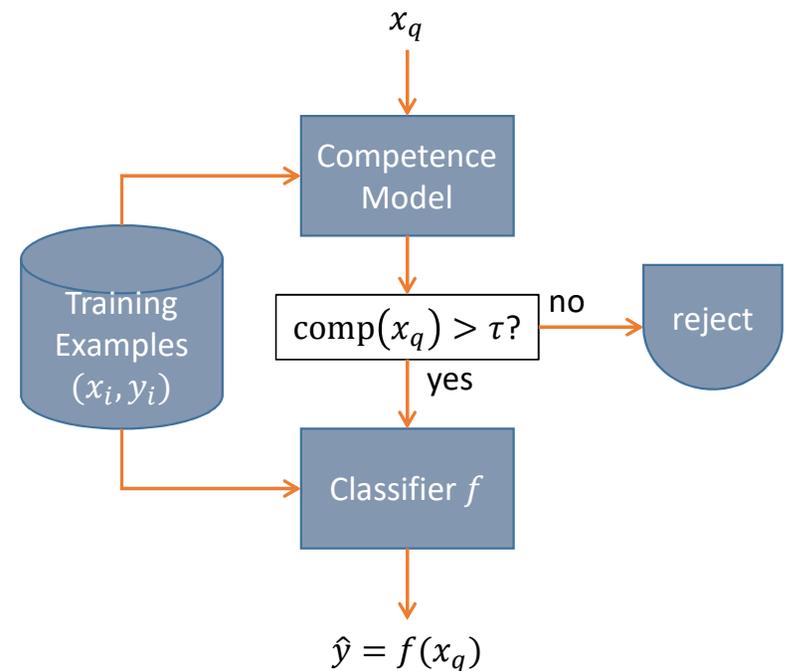
Problem: There are $\approx 76,000$ species of freshwater insects worldwide

- 1,200 species in US
- Field samples may contain other things
 - leaves
 - trash
- Simple estimate of equal error rate for novel classes vs. the 54 classes was 20% (in 2011)
 - classifier is not usable without addressing the novel class problem
- Open Category Problem



Open Set/OOD Classification Problem

- Arises in any application of classification in an open world
 - Novel obstacles in self-driving cars
 - Novel diseases in medical imaging
 - Novel products in online marketplaces
 - Novel cyber attacks
- Claim: Every deployed ML classifier should include a competence model that can detect when new queries are far from the training data



Two Problem Formulations: OOD and Open Category

Out-of-Distribution Problem

- Training:
 - Data: $(x_1, y_1), \dots, (x_N, y_N)$ drawn from D_0
 - $y_i \in \{1, \dots, K\}$
- Testing:
 - Data: Mixture D_m of data from D_0 and D_a
 - $(x, y) \sim D_a$ belong to a **different data set**
- Goal:
 - Given a query x_q , does it belong to D_a or D_0 ?
 - If from D_a , REJECT as alien
 - Else classify using a classifier trained on D_0 data

Novel Category / Open Set Problem

- Training:
 - Data: $(x_1, y_1), \dots, (x_N, y_N)$ drawn from D_0
 - $y_i \in \{1, \dots, K\}$
- Testing:
 - Data: Mixture D_m of data from D_0 and D_a
 - $(x, y) \sim D_a$ belong to **new classes not seen during training (“alien categories”)**
- Goal:
 - Given a query x_q , does it belong to D_a or D_0 ?
 - If from D_a , REJECT as alien
 - Else classify using a classifier trained on D_0 data

OOD and Novel Category Metrics

- **AUROC**: Area under the ROC curve for the binary decision
 - OOD: Domain A vs Domain B
 - Novel Category: Known vs Unknown
 - We treat the anomalies as the positive class
- Detection rate at fixed false alarm rate. **Maximize TPR@10%FPR**
 - Maximize correct OOD/Novel Category detections subject to a constraint that the false positive rate is ≤ 0.10 .
- False alarm rate at fixed missed alarm rate: **Minimize FPR@95%TPR**
 - Detect 95% of OOD/Novel Category examples while minimizing false positives
 - Most relevant to AI Safety and Trustworthy Systems

Outline

- Theoretical Approaches to Anomaly Detection
- Deep Anomaly Detection in Computer Vision
- Anomaly Detection based on Supervised Classifier Logit Scores

Theoretical Approaches to Anomaly Detection

- **Distance-Based Methods**

- Anomaly score

$$A(x_q) = \min_{x \in D} \|x_q - x\|$$

- **Density Estimation Methods**

- Surprise: $A(x_q) = -\log P_D(x_q)$
- Model the joint distribution $P_D(x)$ of the input data points $x_1, \dots \in D$

- **Quantile Methods**

- Find a smooth function f such that $\{x: f(x) \geq 0\}$ contains $1 - \alpha$ of the training data
- Anomaly score $A(x) = -f(x)$

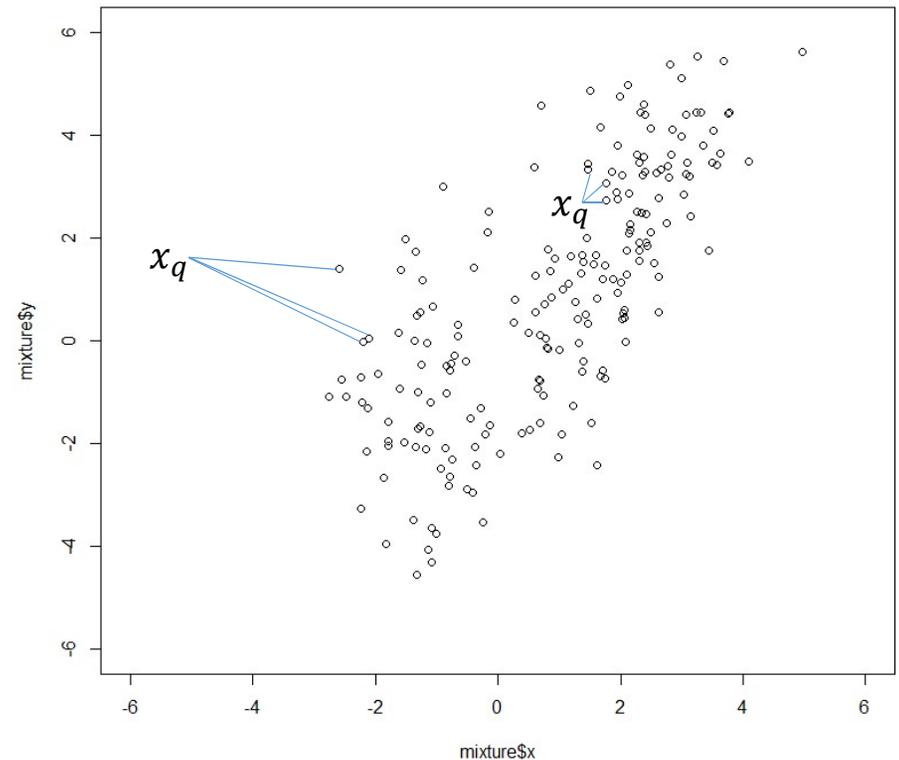
- **Reconstruction Methods**

- Train an auto-encoder: $x \approx D(E(x))$, where E is the encoder and D is the decoder
- Anomaly score

$$A(x_q) = \|x_q - D(E(x_q))\|$$

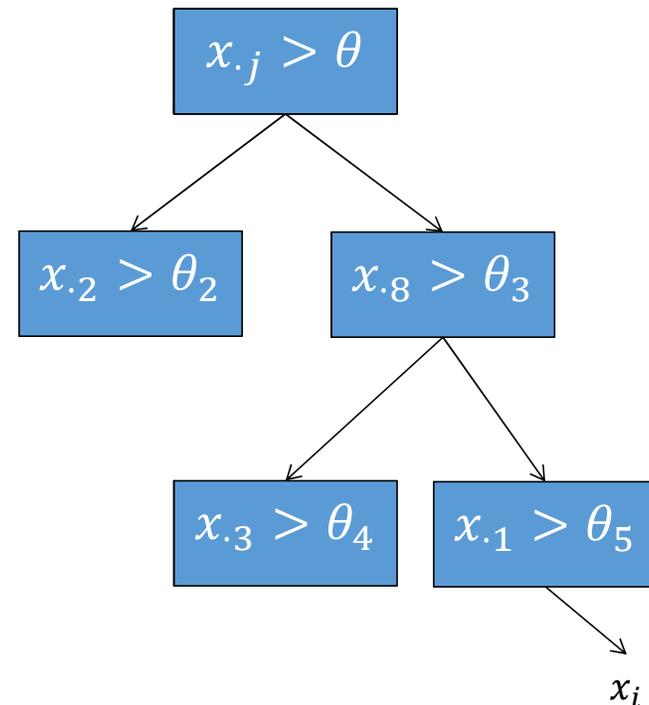
Approach 1: Distance-Based Methods

- Define a distance $d(x_i, x_j)$
- $A(x_q) = \min_{x \in D} d(x_q, x)$
- Requires a good distance metric



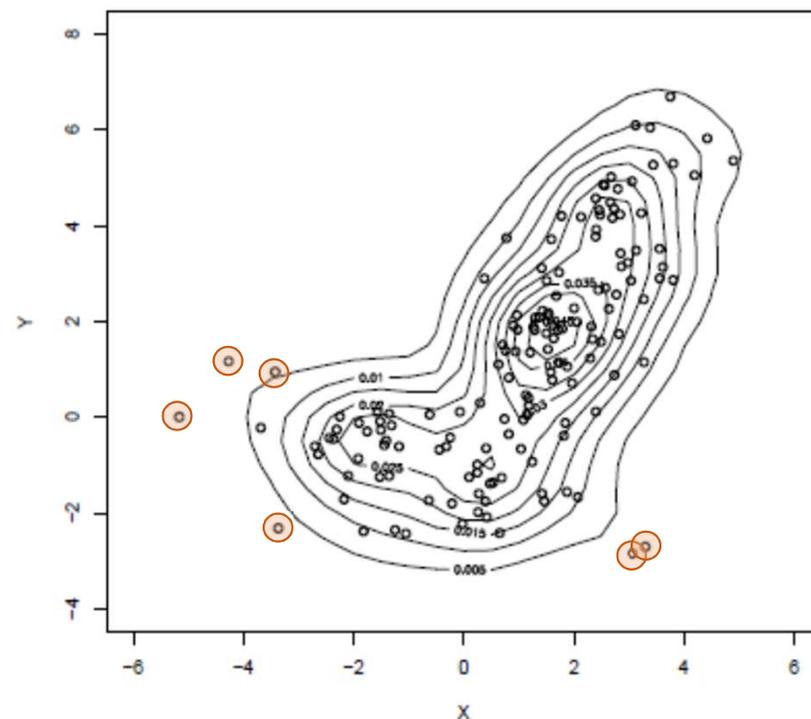
Isolation Forest [Liu, Ting, Zhou, 2011]

- Approximates L1 Distance
 - (Guha, et al., ICML 2016)
- Construct a fully random binary tree
 - choose attribute j at random
 - choose splitting threshold θ uniformly from $[\min(x_j), \max(x_j)]$
 - until every data point is in its own leaf
 - let $d(x_i)$ be the depth of point x_i
- repeat L times
 - let $\bar{d}(x_i)$ be the average depth of x_i
 - $A(x_i) = 2^{-\left(\frac{\bar{d}(x_i)}{r(x_i)}\right)}$
 - $r(x_i)$ is the expected depth



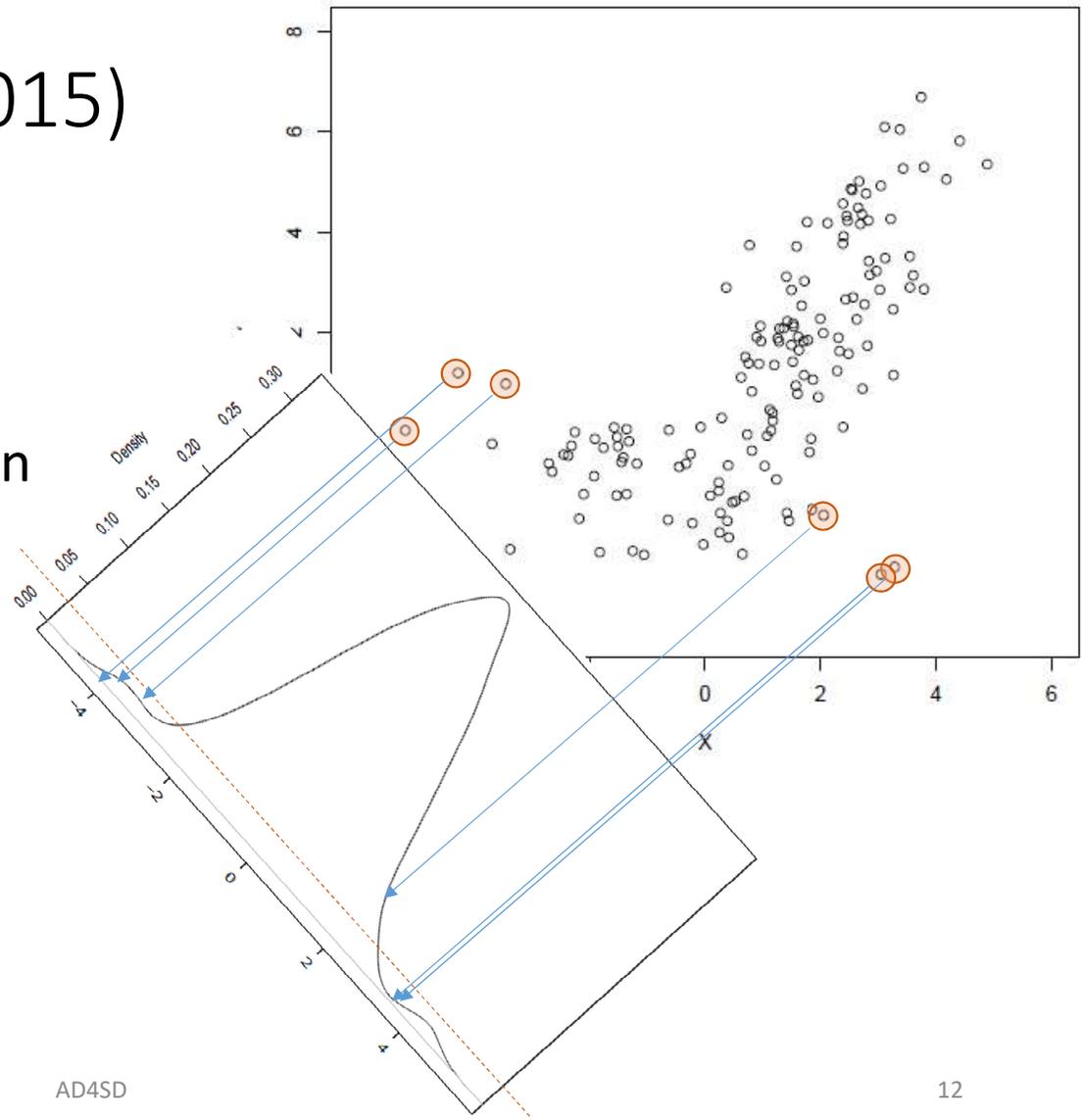
Approach 2: Density Estimation

- Given a data set $\{x_1, \dots, x_N\}$ where $x_i \in \mathbb{R}^d$
- We assume the data have been drawn iid from an unknown probability density: $x_i \sim P(x_i)$
- Goal: Estimate P
- Anomaly Score: $A(x_q) = -\log P(x_q)$
 - “surprisal” from information theory
- Why density estimation?
 - Gives a more global view by combining distances to all data points



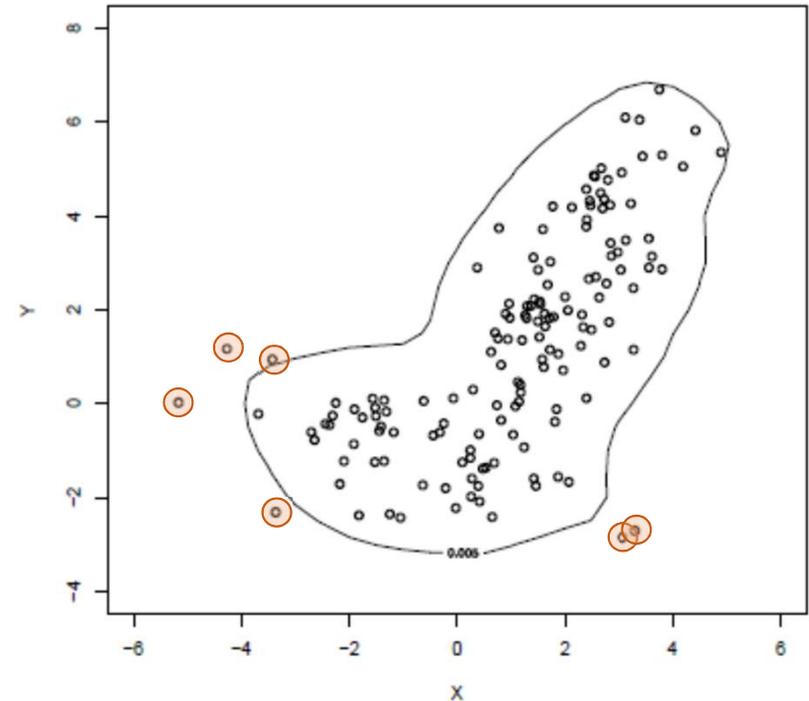
Example: LODA (Pevny, 2015)

- Introduce random projections Π_l into 1-dimensional space
- Fit a density estimator $P_l(\Pi_l(x))$ in each 1-d space
- $A(x) = \frac{1}{L} \sum_{l=1}^L -\log P_l(\Pi_l(x_q))$



Approach 3: Quantile Methods

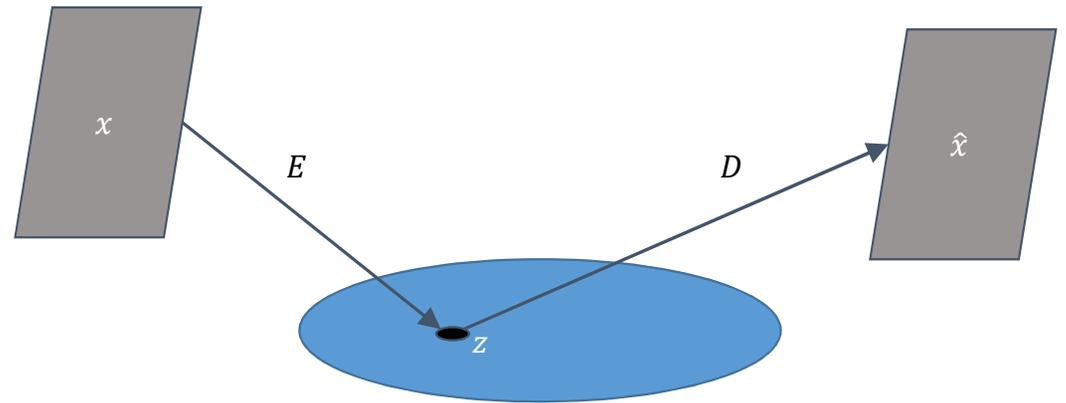
- Vapnik's principle: We only need to estimate the "decision boundary" between nominal and anomalous
- Surround the data by a function f that captures $1 - \epsilon$ of the training data
 - One-Class Support Vector Machine (OCSVM)
 - f is a hyperplane in "kernel space"
 - Support Vector Data Description (SVDD)
 - f is a sphere in "kernel space"
- Issue
 - Need to choose ϵ at learning time rather than run time



Approach 4: Reconstruction Methods

Autoencoders

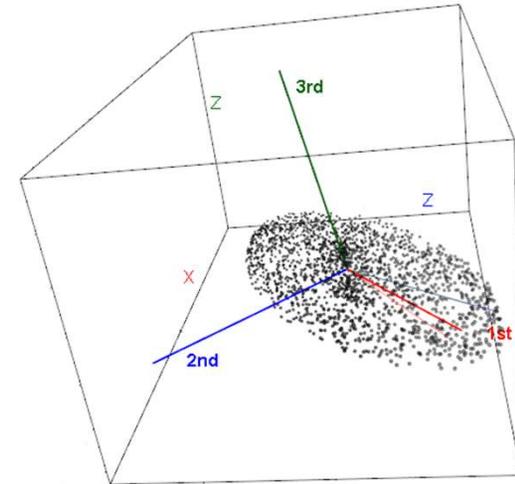
- Encoder: $z = E(x)$
- Decoder: $\hat{x} = D(z)$



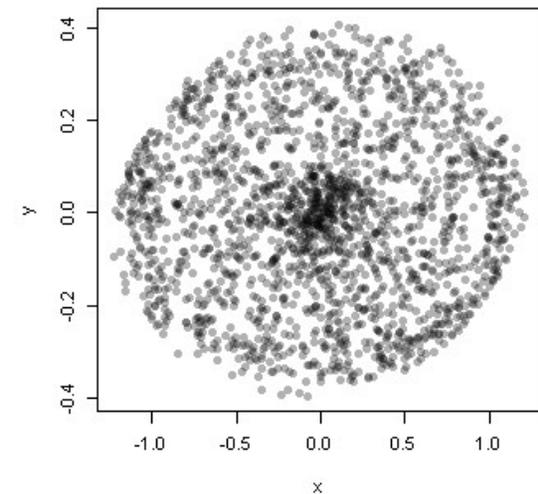
Linear Autoencoder == Principal Component Analysis

- PCA:
 - Let the input dimension be d
 - Choose a latent dimension ℓ
 - Find the $d \times \ell$ matrix W that minimizes the squared reconstruction error
 - $\min_W \sum_i \|x_i - WW^T x_i\|^2$
 - This can be done using the Singular Value Decomposition
 - It can also be viewed as fitting a multi-variate Gaussian to the data and then keeping only the ℓ dimensions of highest variance

PCA applied to an ellipsoidally shaped point cloud



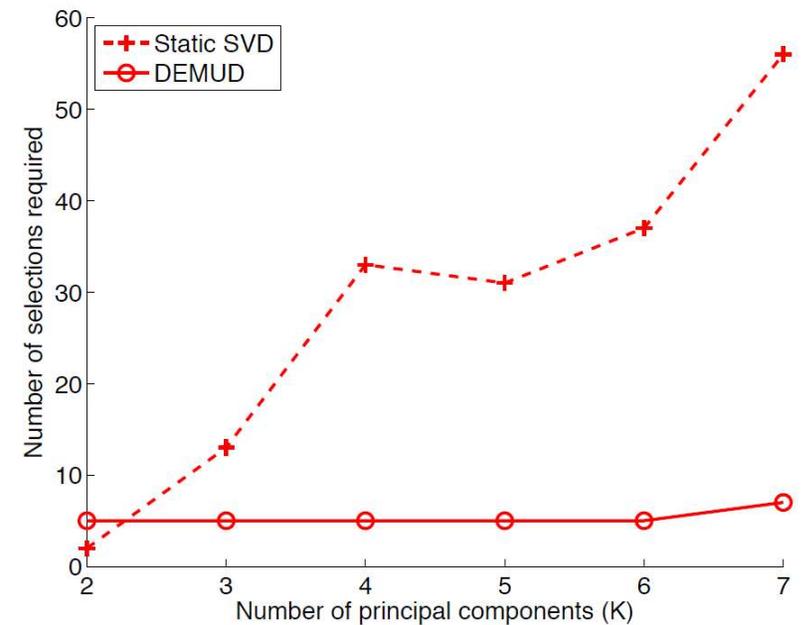
Projection against 1st and 2nd eigen vector



AD4SD

Application: Finding Unusual Chemical Spectra

- NASA Mars Science Laboratory ChemCam instrument
 - Collects 6144 spectral bands on rock samples from 7m distance using laser stimulation
 - Goal: active learning to find interesting spectra
 - DEMUD
 - Incremental PCA applied to samples one at a time
 - Fit only to the samples labeled as “uninteresting” by the user
 - Show the user the most un-uninteresting sample (sample with highest PCA reconstruction error)
 - Rapidly discovers interesting samples
 - Wagstaff, et al. (2013)



(a) Effort required to discover magnesite.

Benchmarking Study

[Andrew Emmott, 2015, 2020]

- Distance-Based Methods
 - k-NN: Mean distance to k -nearest neighbors
 - LOF: Local Outlier Factor (Breunig, et al., 2000)
 - ABOD: kNN Angle-Based Outlier Detector (Kriegel, et al., 2008)
 - IFOR: Isolation Forest (Liu, et al., 2008)
- Density-Based Approaches
 - RKDE: Robust Kernel Density Estimation (Kim & Scott, 2008)
 - EGMM: Ensemble Gaussian Mixture Model (our group)
 - LODA: Lightweight Online Detector of Anomalies (Pevny, 2016)
- Quantile-Based Methods
 - OCSVM: One-class SVM (Schoelkopf, et al., 1999)
 - SVDD: Support Vector Data Description (Tax & Duin, 2004)

Benchmarking Methodology

- Select 19 data sets from UC Irvine repository
- Choose one or more classes to be “anomalies”; the rest are “nominals”
- Manipulate
 - Relative frequency
 - Point difficulty
 - Irrelevant features
 - Clusteredness
- 20 replicates of each configuration
- Result: 11,888 Non-trivial Benchmark Datasets

Analysis of Variance

- Linear ANOVA

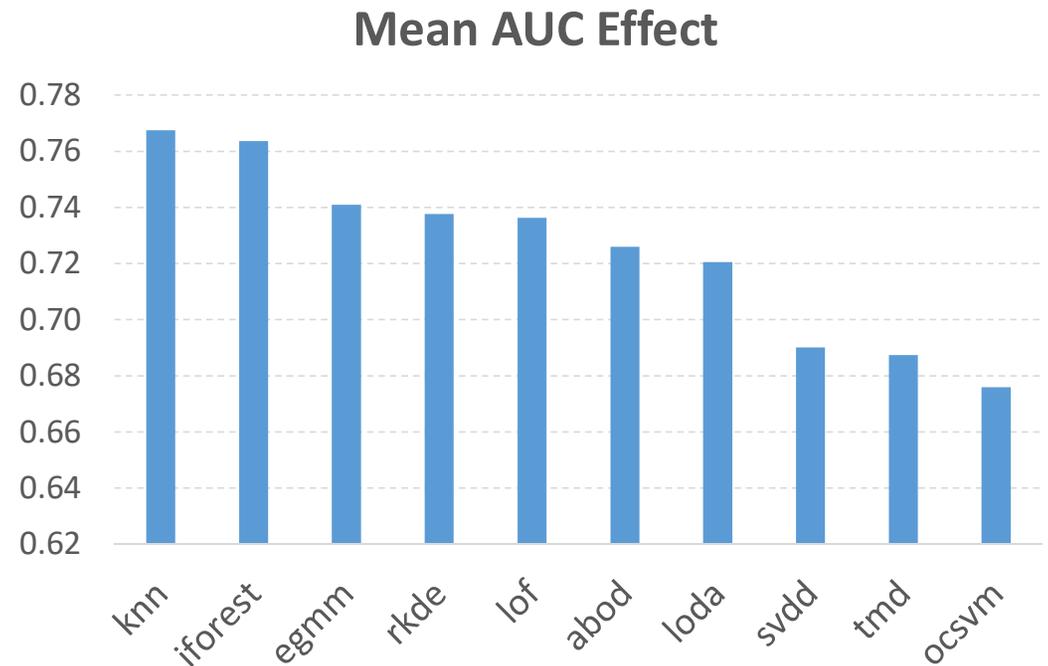
- $\log \frac{AUC}{1-AU} \sim rf + pd + cl + ir + pset + algo$

- rf: relative frequency
 - pd: point difficulty
 - cl: normalized clusteredness
 - ir: irrelevant features
 - mset: "Parent" set
 - algo: anomaly detection algorithm

- Assess the *algo* effect while controlling for all other factors
- *AUC*: area under the ROC curve for the nominal vs. anomaly binary decision

Benchmarking Study Results

- 19 UCI Datasets
- 9 Leading “feature-based” algorithms
- 11,888 non-trivial benchmark datasets
- Mean AUC effect for “nominal” vs. “anomaly” decisions
 - Controlling for
 - Parent data set
 - Difficulty of individual queries
 - Fraction of anomalies
 - Irrelevant features
 - Clusteredness of anomalies
- Baseline method: Distance to nominal mean (“tmd”)
- Best methods: K-nearest neighbors and Isolation Forest
- Worst methods: Kernel-based OCSVM and SVDD



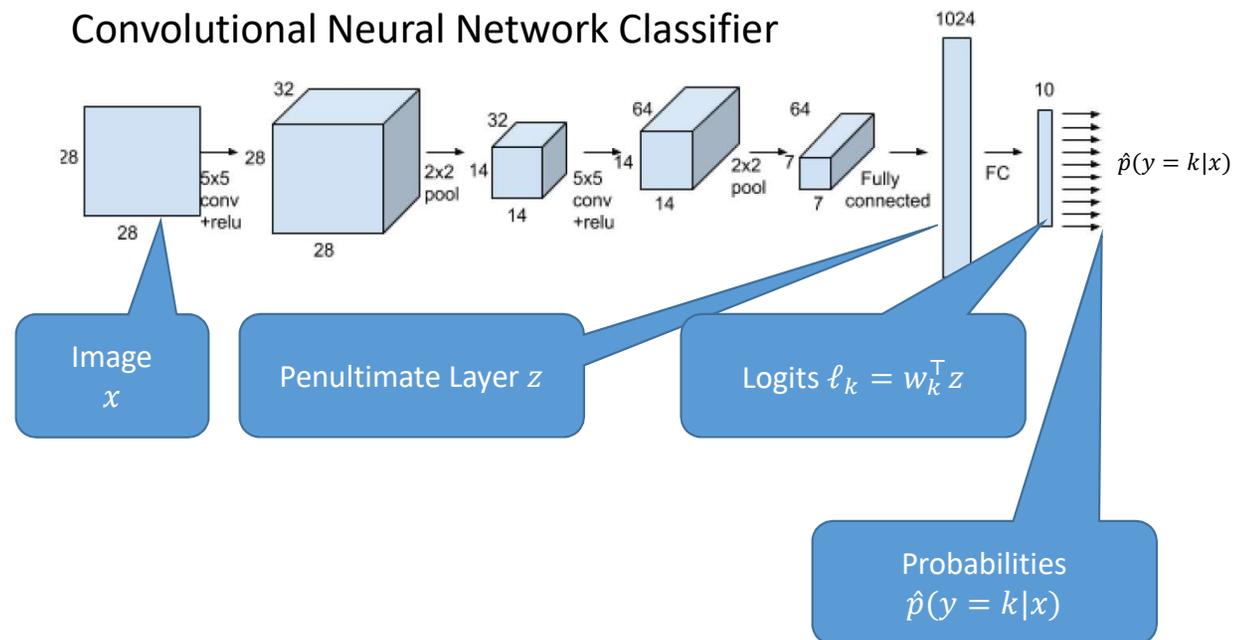
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Deep Anomaly Detection in Image Classification

- Input image x
- Network backbone, also called the “encoder”: $z = E(x)$
- Latent representation z
- Logits $\ell_k = w_k^\top z$
- Predicted probabilities

$$\hat{p}(y = k|x) = \frac{\exp \ell_k(z)}{\sum_{k'} \exp \ell_{k'}(z)}$$



Approach 1: Distance-Based Methods

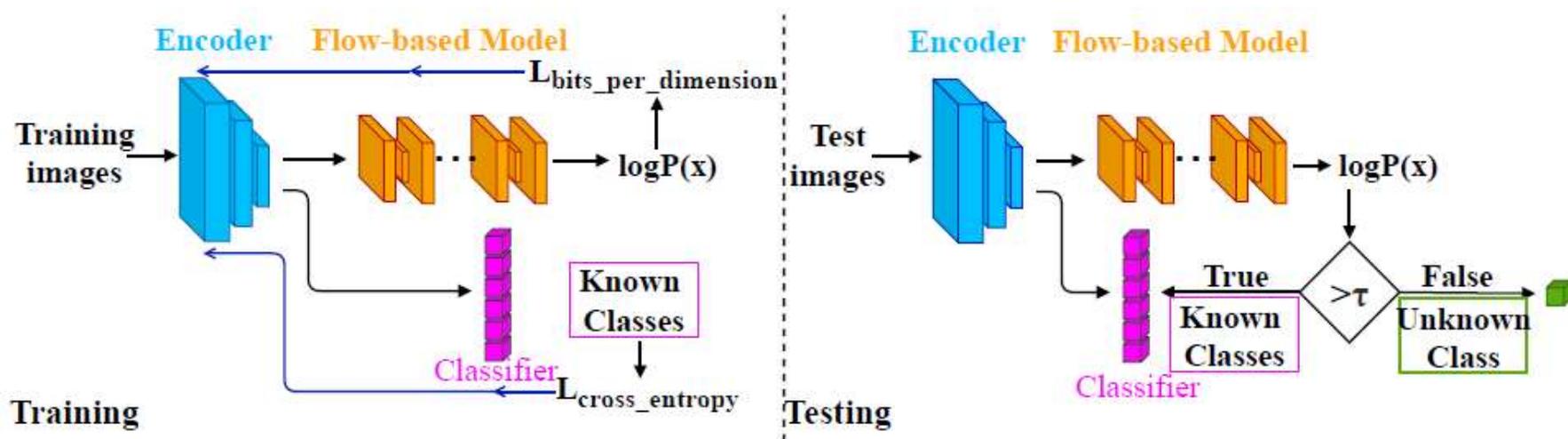
- Using k nearest neighbors in the z space does not work well in our experience

Approach 2: Density Estimation Methods

- Deep Density Estimation: Seeks to model $P(x)$ in the image space
 - These have not worked well
 - Le Lan & Dinh (2021) show that the representation is critical for density estimation. The image space lacks an appropriate “neighborhood structure”
- Methods that fit a classifier and then estimate a density $P(z)$
 - Open Hybrid (Zhang, Li, Guo, Guo, 2020)
 - Mahalanobis (Lee, Lee, Lee, Shin, 2018)

Open Hybrid: Classification + Density Estimation

(Zhang, Li, Guo, Guo, 2020)



- Residual Flow Deep Density Estimator
 - (Chen, Behrmann, Duvenaud, et al. NeurIPS 2019)
- Standard Cross-Entropy Supervised Loss
 - Claim: This helps focus $P(x)$ on relevant aspects of the images
- Anomaly Score: $A(x_q) = -\log P(x_q)$

Open Hybrid Results

- 6 Known and 4 Unknown classes

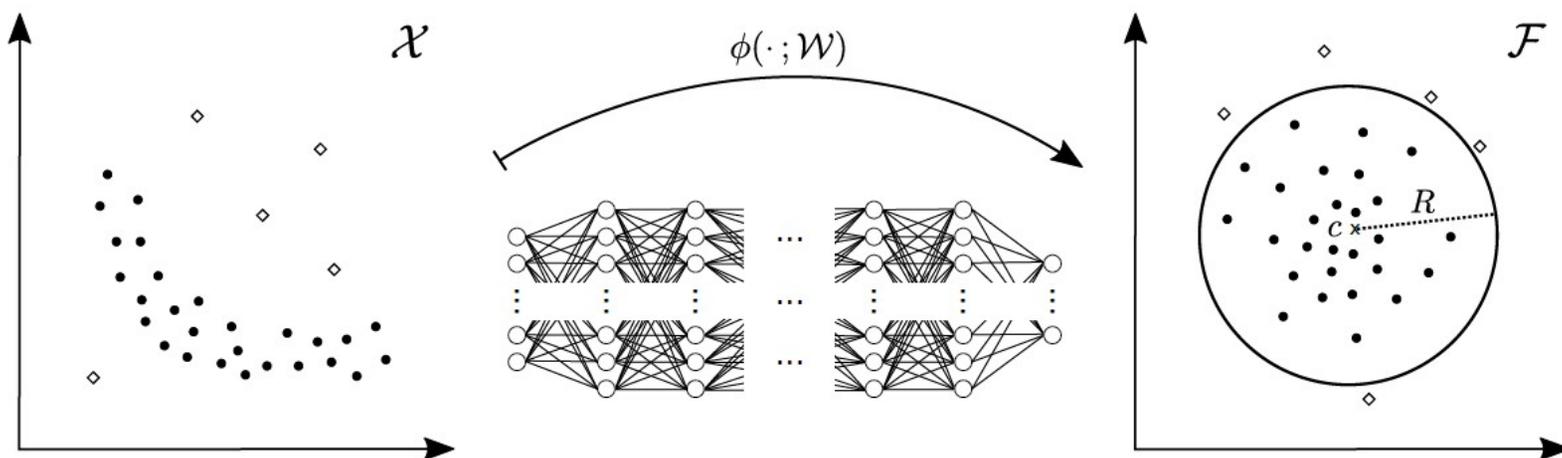
AUC	MNIST	SVHN	CIFAR-10
$1 - \max_k \hat{p}(y = k x_q)$	0.978	0.886	0.677
OpenHybrid: $-\log P(x_q)$	0.995	0.947	0.883

- 4 Known and many unknown classes drawn from CIFAR-100

AUC	CIFAR+10	CIFAR+50
$1 - \max_k \hat{p}(y = k x_q)$	0.816	0.805
OpenHybrid: $-\log P(x_q)$	0.962	0.955

Approach 3: Density Quantile Methods: Deep SVDD

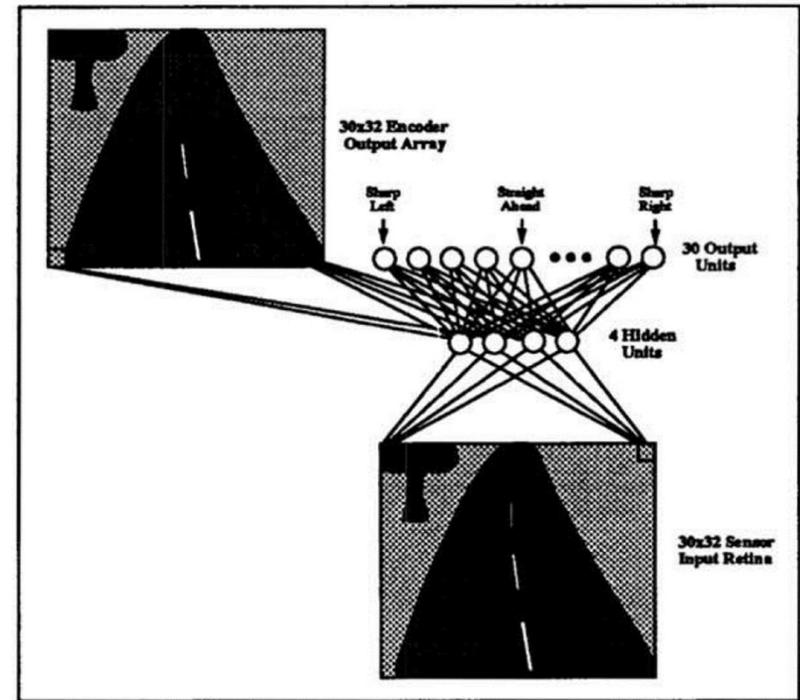
(Ruff, et al. ICML 2018)



- The method is somewhat tricky to work with
 - Set c as the mean of a small set of points passed through the untrained network
 - No bias weights
 - These help prevent “hypersphere collapse”

Approach 4: Reconstruction Methods

- NavLab self-driving van (Pomerleau, 1992)
 - Primary head: Predict steering angle from input image
 - Secondary head: Predict the input image (“auto-encoder”)
 - $A(x_q) = \|x_q - \hat{x}_q\|$
 - If reconstruction is poor, this suggests that the steering angle should not be trusted
- Principle: Anomaly Detection through Failure
 - Define a task on which the learned system should fail for anomalies



Pomerleau, NIPS 1992

Deep Autoencoders

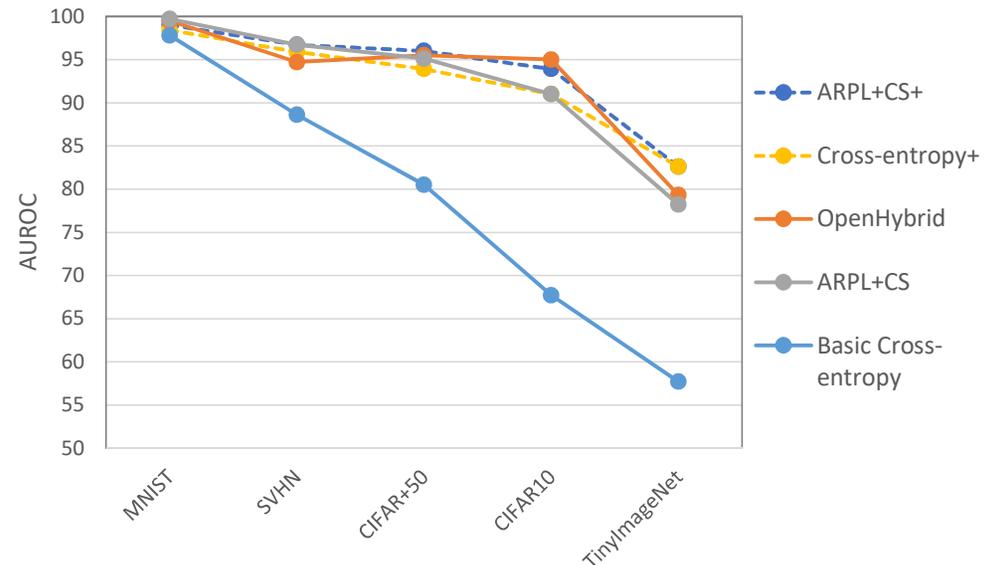
- These have generally not worked well for anomaly detection
- It is difficult to keep the autoencoder from learning to be a general image compression algorithm
 - It doesn't fail on novel images!

Outline

- Theoretical Approaches to Anomaly Detection
- Deep Anomaly Detection in Computer Vision
- **Anomaly Detection based on Supervised Classifier Logit Scores**

A Surprising Finding: Deep Classifiers Achieve Excellent Anomaly Detection

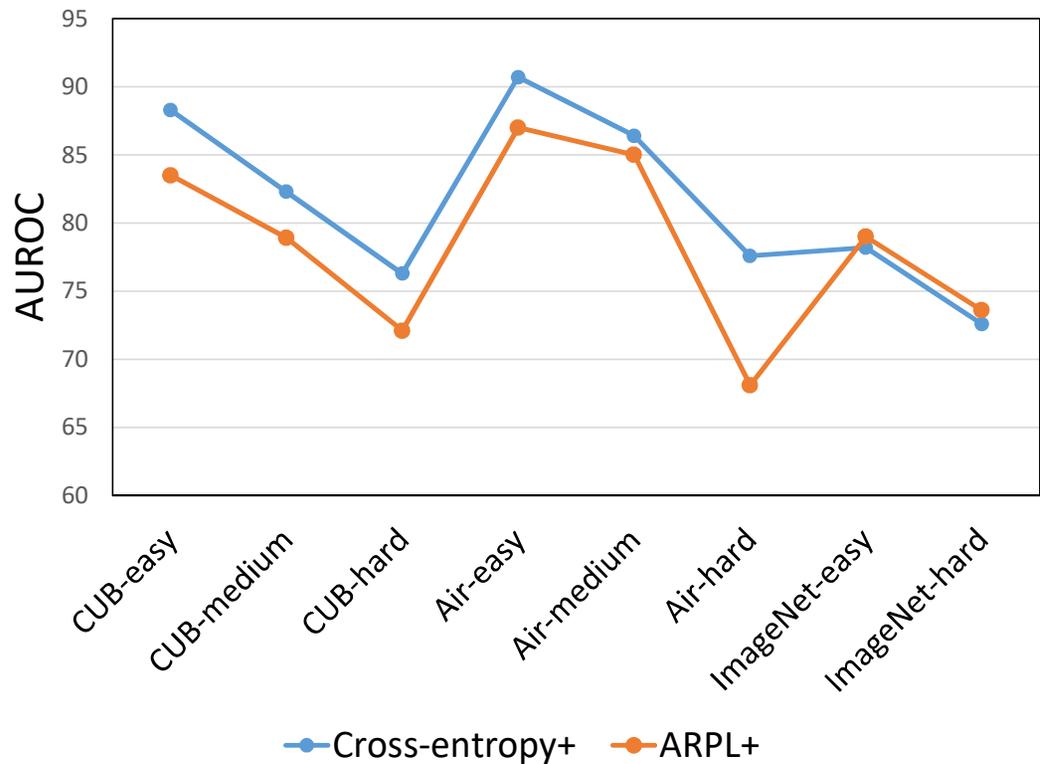
- Vaze, Han, Vedaldi, Zisserman (2020):
“Open Set Recognition: A Good Classifier is All You Need”
 - arXiv 2110.06207
- Carefully train a classifier using the latest tricks
 - Standard cross-entropy combined with the following:
 - Cosine learning rate schedule
 - Learning rate warmup
 - RandAugment augmentations
 - Label Smoothing
- Anomaly score: max logit
 - $-\max_k \ell_k$



Protocol from Neal et al. (2018)

Vaze, et al.: Three Large Open Set Benchmarks

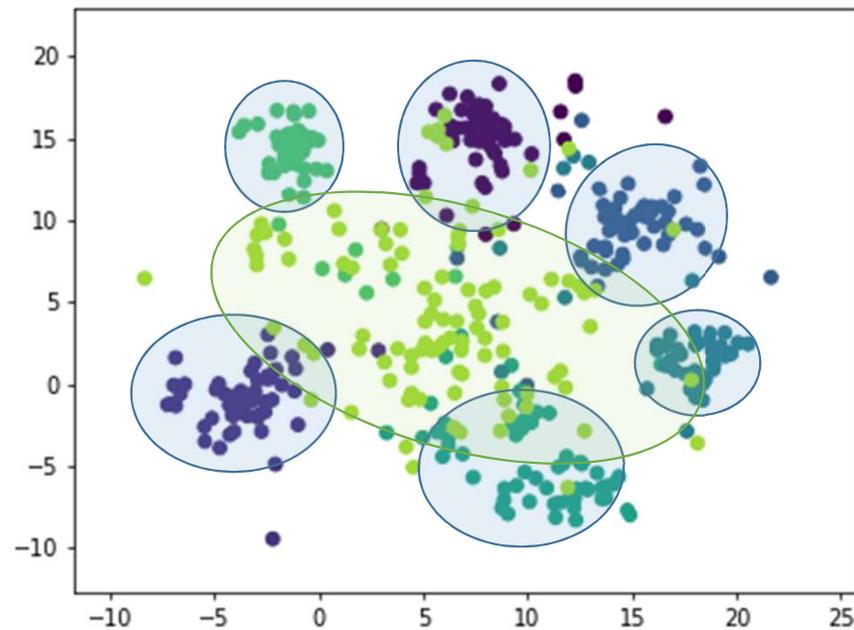
- Novel class difficulty based on semantic distance
 - CUB: Bird species
 - Air: Aircraft
 - ImageNet



Why?

How are open set images represented by deep learning?

- DenseNet with 384-dimensional latent space.
- CIFAR-10: 6 known classes, 4 novel classes
- UMAP visualization
- Light green: novel classes
- Darker greens: known classes
- Note that many novel classes stay toward the center of the space; others overlap with known classes
- Training was not required to “pull them out” so that they could be discriminated



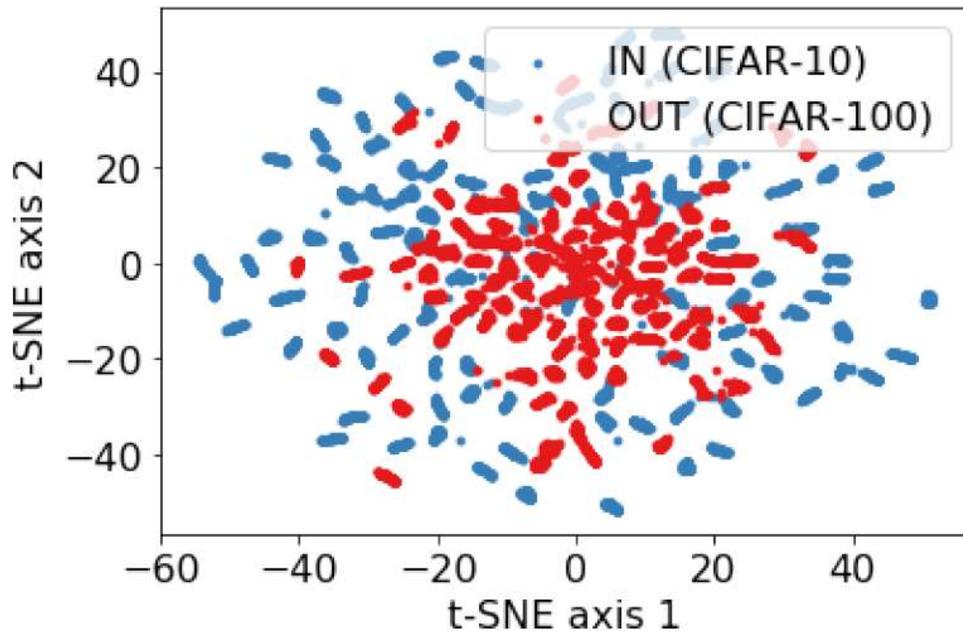
6 Known
Classes

4 Novel
Classes

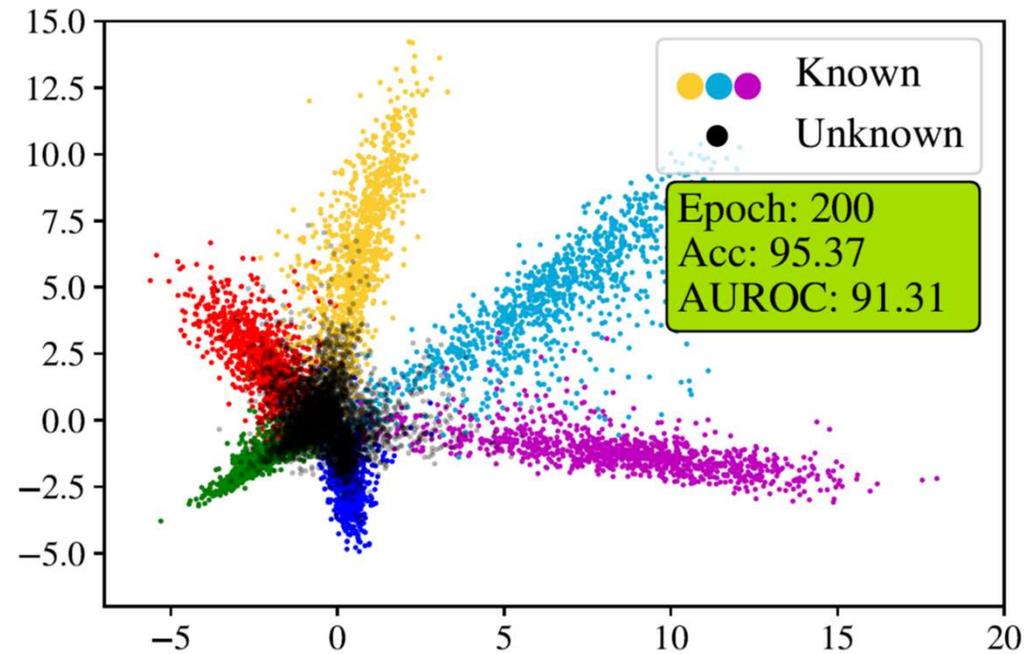
Alex Guyer

Similar Results from Other Groups

t-SNE visualization of features



[Tack, et al. NeurIPS 2020]

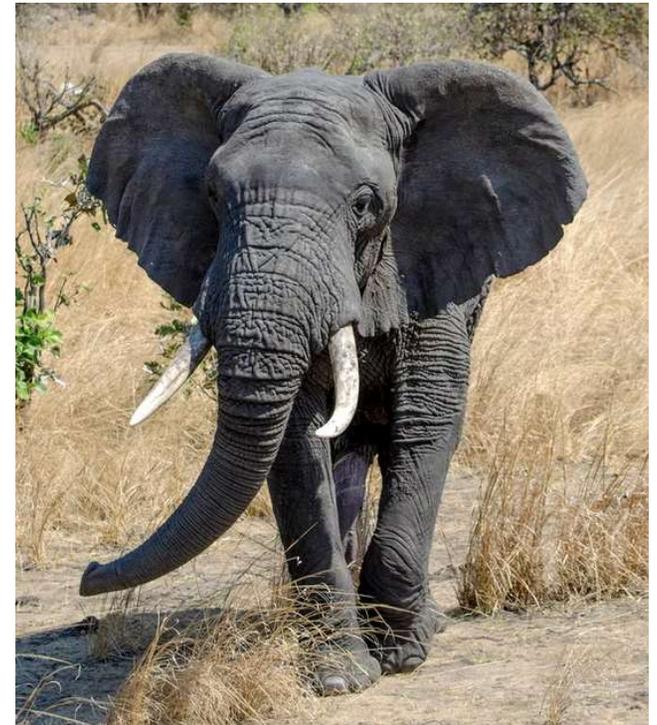


[Vaze, et al. arXiv 2110.06207]

The Familiarity Hypothesis

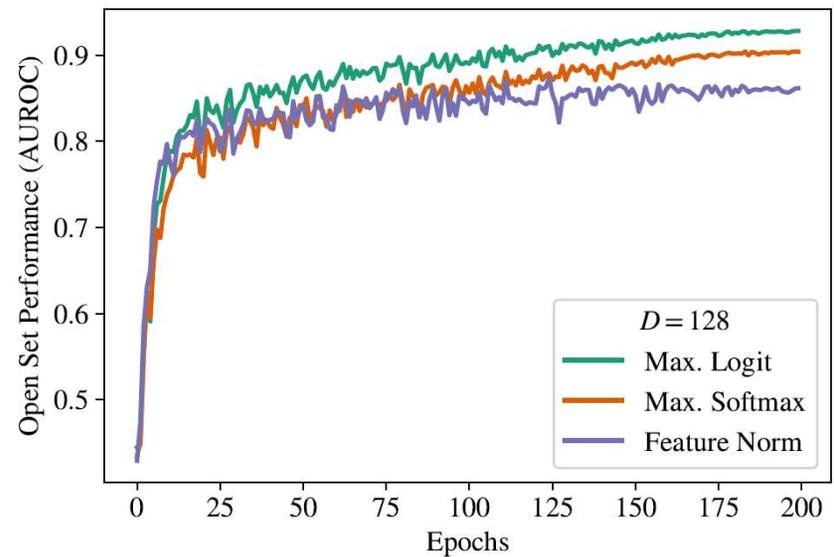
The network doesn't detect novelty, it detects the absence of familiarity

- Convolutional neural network learns “features” that detect image patches relevant to the classification task
- The logit layer weights these features to make the classification decision
- Novel classes activate fewer of these features, so their activation vectors are smaller
- Hypothesis: The networks don't detect that an elephant is novel because of trunk and tusks but because its head doesn't activate known features



Initial Evidence

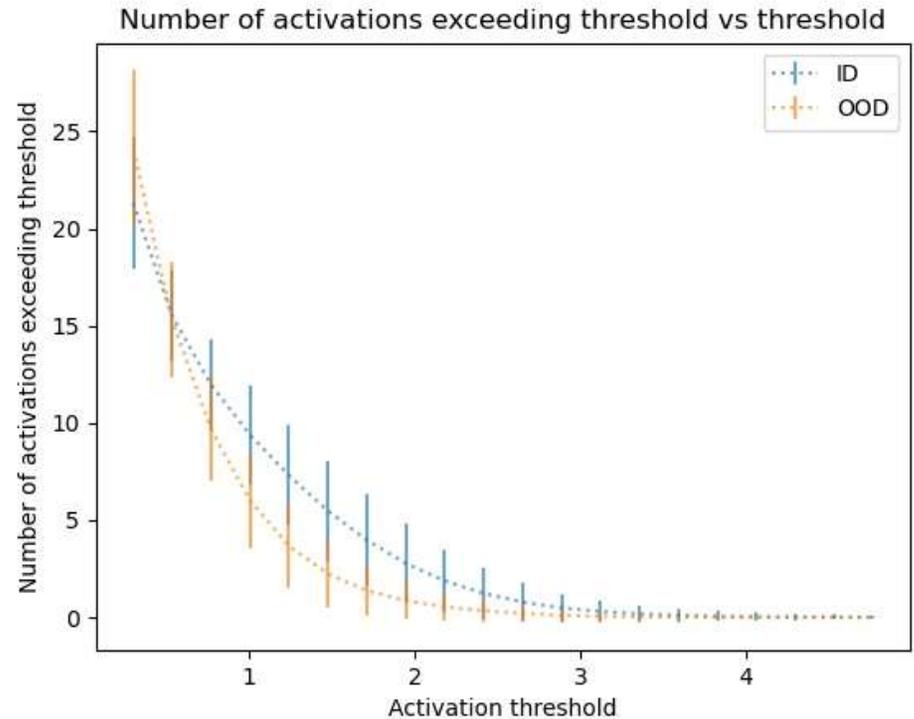
- Maximum logit is better than max softmax probability or the norm of z



Vaze, et al. 2021

Initial Evidence (2)

- CIFAR 10: 6 known classes; 4 novel classes
- DenseNet (z has 324 dimensions)
- Activation threshold θ
- Count number of features whose activation exceeds θ
- OOD images activate fewer features



Alex Geyer (unpublished)

Max Sum of Positive Contributions

- Let w_k be the vector of weights for the logit of class k

$$\ell_k = w_k \cdot z$$

- Define the *contribution* of feature j to the logit for class k

$$c_{jk} = w_{jk} z_j$$

- Sort in ascending order
- Sum of positive contributions

$$s_k = \sum_j \max(0, c_{jk})$$

- Claim: $\max_k s_k$ will be an even better anomaly detector than max logit

Mean and variance of unit contributions:

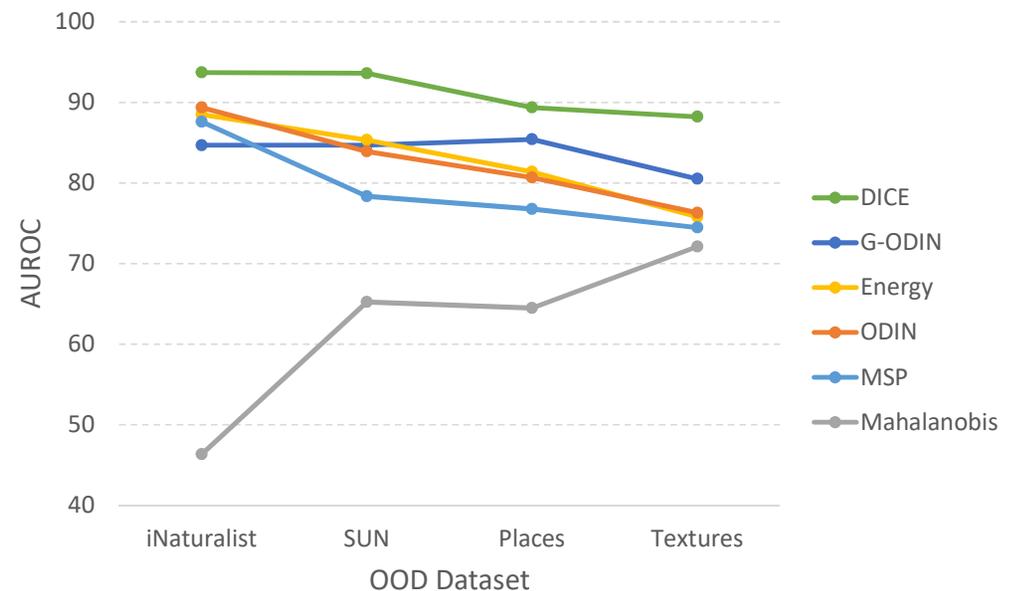


Positive contributions:
Features that predict class k

Negative contributions:
Features that predict other classes

Sun & Li (2021) Did almost this experiment

- DICE anomaly score: sum of the top 10% of the contributions
 - MSP: $\max_k \hat{P}(y = k|x)$
 - Energy: $\log \sum_k \exp \ell_k$
 - ODIN and G-ODIN modify the Softmax
- Sun & Li (2021) “On the Effectiveness of Sparsification for Detecting the Deep Unknowns” arXiv 2111.09805



Can we expect computer vision systems to perceive things they have not been trained on?

- Colin Blakemore and Grahame F. Cooper. “Development of the brain depends on the visual environment.” *Nature* (1970): 477-478.
 - Kittens raised in environments with only horizontal or only vertical lines
 - “They were virtually blind for contours perpendicular to the orientation they had experienced.”



Source: Li Yang Ku

<https://computervisionblog.wordpress.com/2013/06/01/cats-and-vision-is-vision-acquired-or-innate/>

Possible Paths Forward

- Train on as many object categories as possible
 - Training on snakes might allow detection of trunks
- Systematically synthesize “natural parts”
 - Could we synthesize tusks, trunks if we had never seen them before?
 - Train on these to develop feature detectors for them
- Could a system detect “interesting image content” that was not activating learned features?

Summary

Classic Anomaly Detection Methods

- Distance
- Density estimation
- Density quantile estimation
- Reconstruction

Computer Vision Methods

- Well-trained Classifier works as well as deep anomaly detection methods
- Familiarity Hypothesis explains this and suggests improvements

Citations

- Bendale, A., & Boulton, T. (2016). Towards Open Set Deep Networks. In CVPR 2016 (pp. 1563–1572). <http://doi.org/10.1109/CVPR.2016.173>
- Blakemore, C., & Cooper, G. F. (1970). Development of the brain depends on the visual environment. *Nature*, 228(5270), 477–478. <https://doi.org/10.1038/228477a0>
- Breunig, M. M., Kriegel, H., Ng, R. T., & Sander, J. (2000). LOF: Identifying Density-Based Local Outliers. *ACM SIGMOD 2000 International Conference on Management of Data*, 1–12.
- Chen, R. T. Q., Behrmann, J., Duvenaud, D., & Jacobsen, J.-H. (2019). Residual Flows for Invertible Generative Modeling. *ArXiv*, 1906.02735(v1), 1–18. <http://arxiv.org/abs/1906.02735>
- Chen, G., Peng, P., Wang, X., & Tian, Y. (2021). Adversarial Reciprocal Points Learning for Open Set Recognition. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 1–17. <https://doi.org/10.1109/TPAMI.2021.3106743>
- Emmott, A. F., Das, S., Dietterich, T., Fern, A., & Wong, W. (2013). Systematic Construction of Anomaly Detection Benchmarks from Real Data. *KDD 2013 Workshop on Outlier Detection (ODD-2013)*, 6.
- Emmott, A. F. (2020). A Benchmarking Study of Unsupervised Anomaly Detection Algorithms. MS Thesis. School of EECS, Oregon State University.
- Guha, S., Mishra, N., Roy, G., & Schrijvers, O. (2016). Robust Random Cut Forest Based Anomaly Detection On Streams. *Proceedings of The 33rd International Conference on Machine Learning*, 48. <http://jmlr.org/proceedings/papers/v48/guha16.pdf>
- Kim, J., & Scott, C. D. (2012). Robust Kernel Density Estimation. *Journal of Machine Learning Research*, 13, 2529–2565.

Citations (2)

- Kriegel, H.-P., Schubert, M., & Zimek, A. (2008). Angle-based outlier detection in high-dimensional data. *Proceedings of the 14th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD)*, 444–452. <https://doi.org/10.1145/1401890.1401946>
- Larios, N., Soran, B., Shapiro, L., Martínez-Muñoz, G., Lin, J., Dietterich, T. G. (2010). Haar Random Forest Features and SVM Spatial Matching Kernel for Stonefly Species Identification. *IEEE International Conference on Pattern Recognition (ICPR-2010)*.
- Lan, C. Le, & Dinh, L. (2020). Perfect density models cannot guarantee anomaly detection. *ArXiv*, 2012.03808(v1), 1–15. <http://arxiv.org/abs/2012.03808>
- Lee, K., Lee, K., Lee, H., & Shin, J. (2018). A simple unified framework for detecting out-of-distribution samples and adversarial attacks. *Advances in Neural Information Processing Systems*, NeurIPS2018, 7167–7177. <http://arXiv.org/abs/1807.03888>
- Lin, J., Larios, N., Lytle, D., Moldenke, A., Paasch, R., Shapiro, L., Todorovic, S., Dietterich, T. (2011). Fine-Grained Recognition for Arthropod Field Surveys: Three Image Collections. *First Workshop on Fine-Grained Visual Categorization (CVPR-2011)*
- Liu, S., Garrepalli, R., Dietterich, T. G., Fern, A., & Hendrycks, D. (2018). Open Category Detection with PAC Guarantees. *Proceedings of the 35th International Conference on Machine Learning*, PMLR, 80, 3169–3178. <http://proceedings.mlr.press/v80/liu18e.html>
- Liu, S., Garrepalli, R., Hendrycks, D., Fern, A., & Hendrycks, D., Dietterich, T. G. (to appear) PAC Guarantees and Effective Algorithms for Detecting Novel Categories. *Journal of Machine Learning Research*.
- Liu, F. T., Ting, K. M., & Zhou, Z.-H. (2008). Isolation Forest. *2008 Eighth IEEE International Conference on Data Mining*, 413–422. <https://doi.org/10.1109/ICDM.2008.17>
- Lytle, D. A., Martínez-Muñoz, G., Zhang, W., Larios, N., Shapiro, L., Paasch, R., Moldenke, A., Mortensen, E. A., Todorovic, S., Dietterich, T. G. (2010). Automated processing and identification of benthic invertebrate samples. *Journal of the North American Benthological Society*, 29(3), 867-874.

Citations (3)

- Pevný, T. (2015). Loda: Lightweight on-line detector of anomalies. *Machine Learning, November 2014*. <https://doi.org/10.1007/s10994-015-5521-0>
- Pomerleau, D. A. (1993). Input Reconstruction Reliability Estimation. *Proceedings of NIPS 1993*, 279–286.
- Rudd, E. M., Jain, L. P., Scheirer, W. J., & Boulton, T. E. (2017). The Extreme Value Machine. *ArXiv, 1506.06112*, 1–12. <https://doi.org/10.1109/TPAMI.2017.2707495>
- Ruff, L., Vandermeulen, R. A., Shoaib, D., Binder, A., Emmanuel, M., & Kloft, M. (2018). Deep One-Class Classification. *International Conference on Machine Learning (ICML 2018)*, 10.
- Schölkopf, B., Platt, J. C., Shawe-Taylor, J., Smola, A. J., & Williamson, R. C. (2001). Estimating the support of a high-dimensional distribution. *Neural Computation, 13*(7), 1443–1471. <https://doi.org/10.1162/089976601750264965>
- Sun, Y., & Li, Y. (2021). On the Effectiveness of Sparsification for Detecting the Deep Unknowns. *ArXiv, 2111.09805*(v1). <http://arxiv.org/abs/2111.09805>
- Tack, J., Mo, S., Jeong, J., & Shin, J. (2020). CSI: Novelty Detection via Contrastive Learning on Distributionally Shifted Instances. *Advances in Neural Information Processing Systems (NeurIPS 2020)*.
- Tax, D., & Duin, R. (2004). Support vector data description. *Machine Learning*, 45–66. <http://link.springer.com/article/10.1023/B:MACH.0000008084.60811.49>
- Vaze, S., Han, K., Vedaldi, A., & Zisserman, A. (2021). Open-Set Recognition: A Good Closed-Set Classifier is All You Need. *ArXiv, 2110.06207*(v1), 1–23. <http://arxiv.org/abs/2110.06207>
- Wagstaff, K. L., Lanza, N. L., Thompson, D. R., Dietterich, T. G., & Gilmore, M. S. (2013). Guiding Scientific Discovery with Explanations using DEMUD. *AAAI 2013*.
- Zhang, H., Li, A., Guo, J., & Guo, Y. (2020). Hybrid Models for Open Set Recognition. *ArXiv, 2003.12506*(v1), 1–17. <http://arxiv.org/abs/2003.12506>